Homework #14

(due April 25)

You should solve each problem analytically, not using a calculator.

1. A market has two sellers, Firm A and Firm B, who sell related but not identical products. The demand for their products is:

$Q_A = 52 - 3P_A + 2P_B$ (except that Q_A can never be negative)

$Q_B = 52 - 3P_B + 2P_A$ (except that Q_B can never be negative)

The firms have the same cost function: $C(Q) = 16Q$.

Suppose the firms play a game in which each player's action is its price. Firm A chooses $P_A \geq 0$; and Firm B chooses $P_B \geq 0$.

(a) What is Firm A's payoff function, meaning its profit as a function of P_A and P_B? What is Firm B's payoff function?

(b) Calculate the first-order condition for Firm A's optimization problem, treating firm B's action as exogenous.

(c) Does Firm A's optimization problem satisfy the global SOC?

(d) Calculate the solution to Firm A's optimization problem, continuing to treat Firm B's action as if it were exogenous. Remember to account for any boundaries. This is Firm A's best reply function.

(e) Calculate Firm B's best reply function. You can take advantage of symmetry.

(f) Calculate the unique pure Nash equilibria of the game between the two firms. Calculate the firms' prices, quantities, and profits in equilibrium.

(g) Graph the two firms' best reply functions, showing the exact coordinates of any intercepts and the Nash equilibrium.

(h) Draw Firm A's demand curve, given Firm B's action in the Nash equilibrium, showing the exact coordinates of the intercepts. Calculate Firm A's marginal revenue function. Based on the FOC MR=MC, which point on the demand curve should Firm A pick to maximize its profit? Does this match Firm A's action in the Nash equilibrium? Briefly explain, intuitively, why it does or does not match.

(i) Suppose that Firm A moves first, and Firm B sets its price after observing what Firm A does. Set up firm A's optimization problem. Firm B's action should not appear in Firm A's objective function. Calculate Firm A's optimal action, and both firms' profits if Firm A acts optimally. Is it better to move first or second in this game?

2. The scenario is the same, except that Firm A's costs double, so that $C_B(Q) = 16Q$ but $C_A(Q) = 32Q$. The demand functions are unchanged. Repeat parts (a)-(g) from problem #1 (but you can no longer take advantage of symmetry). Also:

(h) Compare the two firms' prices and profits in this problem to their prices and profits in problem (1). (For example, does the doubling of Firm A's costs double its price? Do Firm A's costs affect Firm B's price?) Explain the differences intuitively.